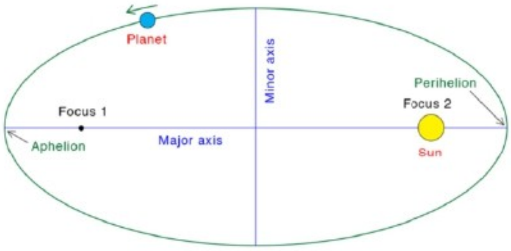
Chapter 7 Formulae List

Gravitation

1. Kepler's laws of plantery action:

First law (law of orbits): *Each planet revolves around sun in elliptical orbit.*

- Sun at one of the foci(at focus 1 or 2)
- Aphelion point: farthest point of planet to sun
- Perihelion point: nearest point of planet to sun
- r_A = distance between sun and Aphelion point
- r_P = distance between sun and Perihelion point



Second law (law of area): *Areal velocity of planets revolving around sun is constant.*

$$\frac{v_P}{v_A} = \frac{r_A}{r_P} \quad \text{or}$$

$$mv_Pr_P = mv_Ar_A \text{ (angular momentum constant)}$$

v_p = velocity at Perihelion point, v_A = velocity at Aphelion point

Third law (law of periods): *Square of time period of revolution equals to cube of semi major axis.*

$$T^2 \propto R^3 \quad \text{or} \quad \frac{T_1}{T_2} = \frac{R_1^3}{R_2^3}$$

T_1 and T_2 is time period of revolution planet 1 and planet 2

R_1 and R_2 is semi major axis for planet 1 and planet 2

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2. Newton's Universal law of Gravitation:

$$F = \frac{Gm_1 m_2}{r^2}$$

m_1, m_2 masses of two objects

r = distance of separation,

G = universal gravitation constant = $6.6.7 \times 10^{-11}\ Nm^2kg^{-2}$

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3. Acceleration due to gravity g:

Weight W= mg

m = mass of object

g = acceleration due to gravity of earth

$g = \frac{GM}{R^2}$

M = mass of earth

For earth $g_e = 9.8 ms^{-2}$

R = Radius of earth

For Moon: $g_M = \frac{GM_M}{R_M^2} = \frac{g_e}{6} = 1.71 ms^{-2}$

Variation of g with height h:

- When height h is comparable to radius of earth R

$$g' = g\left(\frac{R}{R+h}\right)^2$$

- When height h is very less than radius of earth R

$$g' = g\left(1 - \frac{2h}{R}\right)$$

With height the acceleration due to gravity decreases (g' < g)

Decrease in value of g:

$$g' - g = \frac{2hg}{R}$$

Fractional decrease in value of g:

$$\frac{g' - g}{g} = \frac{2h}{R}$$

Percentage decrease in value of g:

$$\frac{g' - g}{g} \times 100 = \frac{2h}{R} \times 100$$

Variation of g with depth d:

$$g' = g\left(1 - \frac{d}{R}\right)$$

With depth the acceleration due to gravity decreases ($g' < g$)

Decrease in value of g:

$$g' - g = \frac{dg}{R}$$

Fractional decrease in value of g:

$$\frac{g' - g}{g} = \frac{d}{R}$$

Percentage decrease in value of g:

$$\frac{g' - g}{g} \times 100 = \frac{d}{R} \times 100$$

Graph of variation of gravity with depth, height, at surface and at the centre of earth:

i) At the centre of earth $g = 0$

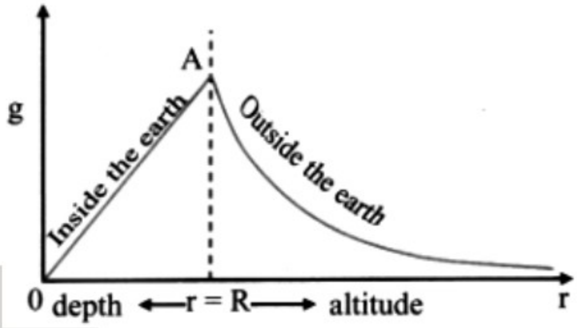
ii) With depth d

$$g' \alpha (R-d)^2$$

iii) At surface g' is maximum $g' = g$

iv) With altitude or height h

$$g' \propto \left(\frac{1}{R+h}\right)^2$$



Variation of g due to shape of earth:

Value of g increases as we go from equator to poles

(gravity at poles) $g_p > g_e$ (gravity at equator)

$$g_p - g_e = 0.034 ms^{-2}$$

$$g_p - g_e = R\omega^2$$

(Radius of earth at equator) $R_e - R_p$ (Radius of earth at poles) = 21km

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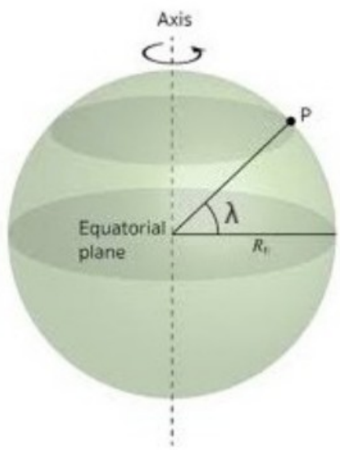
Variation of g due to rotation of earth:

$$g' = g - R\omega^2 \cos^2\lambda$$

ω = angular velocity of rotation of earth

λ = latitude at a place



i) At equator λ = 0

ii) At poles λ = 90°

The acceleration due to gravity decreases on account of rotation of earth.

The acceleration due to gravity increases with latitude of the place

The percentage change in the weight of the body when taken from poles to equator:

$$\frac{mg' - mg}{mg} \times 100 = \frac{R\omega^2}{R} \times 100$$

Intensity of gravitational field (I):

$$I = \frac{F}{m_o} = \frac{-GM}{x^2} = a$$

$$\vec{I} = \frac{-GMm_o}{x^2}\hat{x} \quad \text{and} \quad F = \frac{-GMm_o}{x^2}$$

a = acceleration produced in the body

F = Gravitational force of attraction

M= mass of object

m_o = test mass or unit mass

Units: S.I = ms^{-2} or Nkg^{-1}

C.g.s = cms^{-2} or dyne gm^{-1}

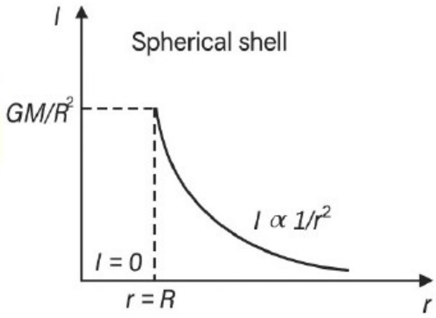
Divisional Formulae: $M^0L^1T^{-2}$

Intensity of gravitational field due to hollow spherical shell:

i) Inside I = 0

ii) At surface $I = \frac{GM}{R^2}$

iii) Outside $I \propto \frac{1}{R^2}$

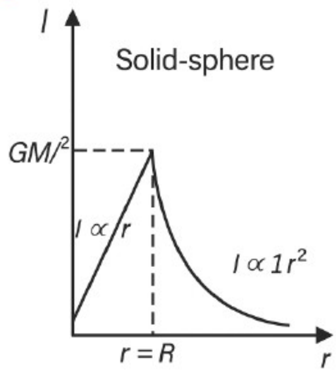


Intensity of gravitational field due to solid sphere

i) Inside $I \propto r$

ii) At surface $I = \frac{GM}{R^2}$

iii) Outside $I \propto \frac{1}{R^2}$



Gravitational Potential (V_P):

$V_P = \frac{W}{m_o} = -\frac{GM}{r}$ - sign is due to work done against gravity

At infinity ($r = \infty$) gravitational potential $V_P = 0$ (maximum value)

At the centre of earth gravitational potential

$V = -\frac{3GM}{2R}$ (minimum value)

At the surface of earth gravitational potential $V = -\frac{GM}{R}$

Gravitational potential due to solid sphere of mass M, Radius R, at a point inside the solid sphere at a distance r from centre of sphere

$V_P = -\frac{GM(3R^2 - r^2)}{2R^2}$

Units: S.I = Jkg^{-1} c.g.s = ergs gm^{-1}

Dimensional formula: $M^0L^2T^{-2}$

Gravitational Potential Energy (U):

$U = W = -\frac{GMm}{r}$

$U = V_P \times m$ at $r = \infty$, $U = 0$ (maximum value)

at centre of earth, $U = -\frac{3GMm}{5R}$ (minimum value)

at surface of earth, $U = -\frac{GMm}{R}$

Change in gravitational potential energy when a body of mass m is moved from a point having distance r_1 to another point of distance r_2

$\Delta U = -GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

Change in gravitational potential energy when a body of mass m is moved from surface of earth to a height h

$\Delta U = mgh$

Relation between Gravitational potential V and Gravitational field intensity I: $I = \frac{-dV}{dr}$

Satellite:

i) Orbital speed: v_o

The minimum speed required to put satellite into its orbit

$$v_o = \sqrt{\frac{GM}{r}} = R\sqrt{\frac{g}{R+h}}$$

R = radius of earth, r = R+h height of satellite from centre of earth

Near the surface of earth $v_o = \sqrt{gR} = 7.92 \text{ kms}^{-2}$

ii) Time period of satellite: T

The time taken to complete one revolution around earth

$$T = \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}}$$

The time taken to complete one revolution around earth near the surface of earth $T = 2\pi\sqrt{\frac{R}{g}} = 5.08 \times 10^3$ sec. = 84.6 minutes

iii) Altitude or height of satellite: h

$$h = \left[\frac{T^2R^2g}{4\pi^2}\right]^{1/3} - R$$

iv) Angular momentum: L

$L = [m^2GMr]^{1/2}$

v) Energy of orbiting satellite: E

Potential energy $U = -\frac{GMm}{r}$

Kinetic energy $K = \frac{GMm}{2r}$

Total mechanical energy $E = U + K = \frac{-GMm}{2r}$

For satellite orbiting close to surface of earth $E = \frac{-GMm}{2R}$

vi) Binding Energy of satellite:

The energy required to remove satellite from its orbit

Binding energy = $- E = \frac{GMm}{2R}$

Escape speed: V_e

The minimum speed to escape from earth's gravitational field

$v_e = \sqrt{\frac{2GM}{R}} = 11.2\ kms^{-1}$

If a body is projected with speed v ($v > v_e$) then the body will move in the interstellar space with speed $v' = \sqrt{v^2 - v_e^2}$