Chapter 6 Formulae List

Motion of System of Particles and Rigid Body

1. Centre of Mass (CM)

- **Definition:** A Point at which the entire mass of a system can be considered to be concentrated.

- **Position of CM for discrete particles:**
When a body can be assumed to made up of particles like m_1, m_2, m_3 and so on its termed as discrete

$\vec{R}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$ or $\vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$ (for 2 particle system)

- **CM for continuous body:**
When a body cannot be assumed to be made up of particles, rather it is supposed to continuity of small mass dm, is termed as continuous body.

$\vec{R}_{cm} = \frac{1}{M}\int \vec{r}\, dm$

2. Linear Momentum of System P

- Total momentum of system of n particles:

$\vec{P}_{total} = \sum m_i \vec{v}_i$ or $\vec{P}_{total} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \text{-------} + m_n\vec{v}_n$

Units: Kilogram meter per second $kgms^{-1}$ (S.I)

Dimensional formula: $M^1L^1T^{-1}$

- **Conservation of linear momentum:**

  If no external force acts on system(isolated system), total momentum remains constant:

  $\vec{P}_{total} = \text{constant}$

---

3. Motion of Centre of Mass

- Velocity of CM:

  $\vec{V}_{cm} = \frac{d\vec{R}cm}{dt} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$

- Acceleration of CM:

  $\vec{A}_{cm} = \frac{d\vec{V}cm}{dt} = \frac{\sum m_i \vec{a}_i}{\sum m_i}$

  - Total force of CM:

    $\vec{F}_{external} = m_1\vec{a}_1 + m_2\vec{a}_2 + \text{----------} = M\vec{A}_{cm}$

---

4. Angular Momentum L

- **For a particle:**

  $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

- **For a rigid body rotating about axis:**

  $L = I\omega$ $\quad$ I→ Moment of inertia, ω → angular velocity

- ***Conservation:*** *If net external torque = 0 → angular momentum constant*

  Units: *Kilogram meter square per second* kgm^2s^{-1} *(S.I)*

  Dimensional formula: $M^1L^2T^{-2}$

- ***Geometrical meaning of angular momentum:***

  $[\vec{L}] = 2m\left[\frac{d\vec{A}}{dt}\right]$ where A is area swept

5. Torque τ

- *Torque (τ) is rotational motion equivalent of force in linear motion*

  $\vec{\tau} = \vec{r}\times\vec{F}$ or $\tau = r F \sin\theta$

- Relation with angular acceleration and angular momentum:

  $\vec{\tau} = I\,\vec{\alpha}$ $\tau = \frac{dL}{dt}$

  Units: *Newton meter* Nm *(S.I)*

  Dimensional formula: $M^1L^2T^{-2}$

6. Equations of Rotational motion:

i) $\omega = \omega_o + \alpha t$ — ω_o *initial angular velocity*

ii) $\theta = \omega_o t + \frac{1}{2}\alpha t^2$ — ω *final angular velocity*, α *angular acceleration*

iii) $\omega^2 - \omega_o^2 = 2\alpha\,\theta$ — θ *angular displacement*, t *time*

Revolution per minute (Rpm) into radian second^{-1} (rads^{-1})

$$n \text{ rpm} = \frac{2\pi n}{60} \text{ rad s}^{-1}$$

7. Power P

$P = \tau\,\omega$ (for rotational motion)

$P = Fv$ (for translational motion)

Units: Watts (S.I)

Dimensional formula: $M^1L^2T^{-3}$

8. Conditions for Equilibrium of a Rigid body :

for a rigid body to be in equilibrium, two conditions must be met:

1. **Translational Equilibrium:** The vector sum of all forces acting on the body must be zero ($\Sigma F = 0$). i.e. the body has no linear acceleration.
2. **Rotational Equilibrium:** The sum of all moments (torques) about any point must be zero ($\Sigma\tau = 0$). i.e. the body experiences no rotational acceleration.

Types of Translational Equilibrium:

1. Static equilibrium 2. Dynamic equilibrium

Types of static equilibrium i) Stable equilibrium

ii) Unstable equilibrium

iii) Neutral equilibrium

Partial Equilibrium:

If body is in Translational Equilibrium but not in Rotational Equilibrium or vice versa

9. Moment of inertia (I):

- **Definition:** Resistance of body to rotation (rotational inertia) about axis

  Note: moment of inertia in rotational motion is equivalent to mass in linear motion

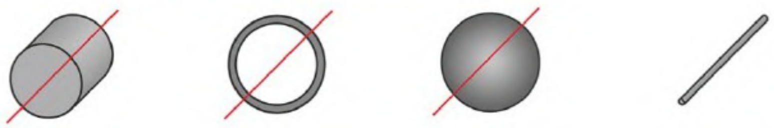
  or

  It is the sum of product of masses of the particles of the body and the respective distances from the axis of rotation

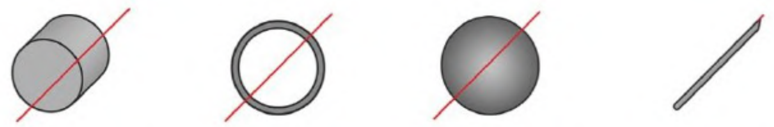
  $I=\sum m_i r_i^2$ (body consists discrete particles),

  $I=\int r^2 dm$ (continuous body)

  Moment of inertia of various shapes:



$I=\frac{1}{2}MR^2$ $\quad I=MR^2$ $\quad I=\frac{2}{5}MR^2$ $\quad I=\frac{1}{12}ML^2$



$I=\frac{1}{4}MR^2$ $\quad I=MR^2$ $\quad I=\frac{2}{3}MR^2$ $\quad I=\frac{1}{3}ML^2$

$I=\frac{1}{12}+ML^2$

- **Radius of Gyration K or R:** The distance of centre of mass from axis of rotation

$$K = \sqrt{\frac{r_1^2 + r_2^2 - - - \mp r_n^2}{n}}$$

Units: meter (S.I)

Dimensional formula: $M^0L^1T^0$

- ***Parallel axis theorem:***

$$I = I_{cm} + Md^2$$

- ***Perpendicular axis theorem:***

$$I_z = I_x + I_y$$

- ***Rotational kinetic energy:***

$KE_{rot} = \frac{1}{2} I\omega^2$ $KE = \frac{1}{2} mv^2$ (Linear motion)

9. Analogy between Linear and Rotational/angular motion

- ***Relation between Rolling motion and linear motion:***

Linear Motion		Rotational Motion	
Position	x	θ	Angular position
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations	$x=\bar{v}+d$	$\theta=\bar{\omega}t$	Motion equations
	$v=v_i+\frac{1}{2}at^2$	$\theta=\omega_i t+\alpha t$	
	$x=v_i t+\frac{1}{2}att^2$	$\theta=\omega_i t+\frac{1}{2}\alpha t^2$	
	$v_f^2=v_i^2+2as$	$\omega^2=\omega_i^2+2\alpha\theta$	
Mass (linear inertia)	m	$\tau=I$	Moment of inertia
Newton's second law	$F=mv$	$\tau=I\alpha$	Angular second law
Momentum	$P=mv$	$L=I\omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	Kinetic energy
Power	Fv	$\tau\omega$	Power