SIMILAR TRIANGLES

6 CHAPTER

CONTENTS

CONCEPT OF SIMILARITY

Geometric figures having the same shape but different sizes are known as similar figures. Two congruent figures are always similar but similar figures need not be congruent.

Illustration 1 :

Any two line segments are always similar but they need not be congruent. They are congruent, if their lengths are equal.

Illustration 2 :

Any two circles are similar but not necessarily congruent. They are congruent if their are equal.

Illustration 3 :

(i) Any two square are similar (see fig. (i))

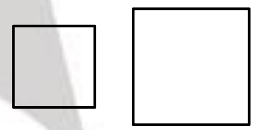


Fig.(i)

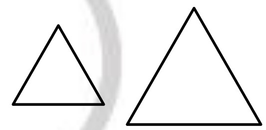


Fig.(ii)

(ii) Any two equilateral triangles are similar (see fig. (ii))

SIMILAR POLYGONS

Definition

Two polygons are said to be similar to each other, if

(i) their corresponding angles are equal, and

(ii) the lengths of their corresponding sides are proportional.

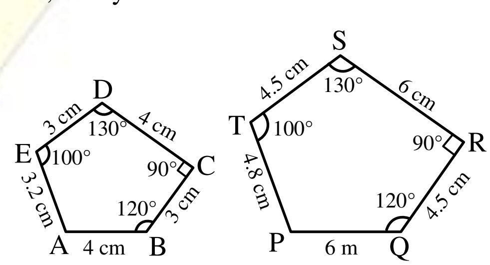
If two polygons ABCDE and PQRST are similar, then from the above definition it follows that :

Angle at A = Angle at P, Angle at B = Angle at Q, Angle at C = Angle at R, Angle at D = Angle at S, Angle at E = Angle at T

and, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DE}{ST}$

If two polygons ABCDE and PQRST, are similar, we write ABCED ~ PQRST.

Here, the symbol '~' stands for is similar to.



SIMILAR TRIANGLE AND THEIR PROPERTIES

Definition

Two triangles are said to be similar, if their

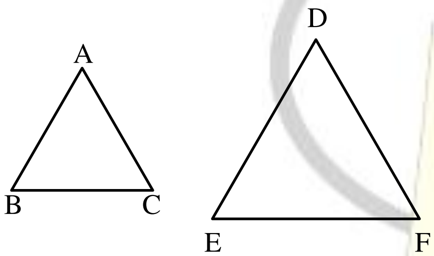
(i) corresponding angles are equal and,

(ii) corresponding sides are proportional.

Two triangles ABC and DEF are similar, if

(i) $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and,

(ii) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



SOME BASIC RESULTS ON PROPORTIONALITY

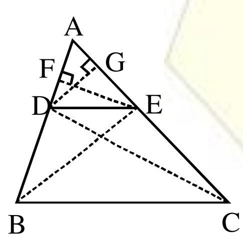
Basic Proportionality Theorem or Thales Theorem

If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

Given : A triangle ABC in which DE ∥ BC, and intersects AB in D and AC in E.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE, CD and draw EF ⊥ BA and DG ⊥ CA.



Proof : Since EF is perpendicular to AB. Therefore, EF is the height of triangles ADE and DBE.

Now, $\text{Area}(\Delta ADE) = \frac{1}{2}(\text{base} \times \text{height}) = \frac{1}{2}(AD.EF)$

and, $\text{Area}(\Delta DBE) = \frac{1}{2}(\text{base} \times \text{height}) = \frac{1}{2}(DB.EF)$

$$\therefore \frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta DBE)} = \frac{\frac{1}{2}(AD.EF)}{\frac{1}{2}(DB.BF)} = \frac{AD}{DB} \quad .... \text{(i)}$$

Similarly, we have

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta DEC)} = \frac{\frac{1}{2}(AE.DG)}{\frac{1}{2}(EC.DG)} = \frac{AE}{EC} \quad .... \text{(ii)}$$

But, ΔDBE and ΔDEC are on the same base DE and between the same parallels DE and BC.

$\therefore$ Area (ΔDBE) = Area (ΔDEC)

From (i), (ii) and (iii), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Since, AD and DB are parts of AB and whereas AE and EC are parts of AC,

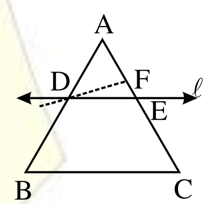
$\therefore$ D and E divide the sides of AB and AC in the same ratio.

Converse of Basic Proportionality Theorem

If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Given : A ΔABC and a line l intersecting AB in D and AC in E, such that $\frac{AD}{DB} = \frac{AE}{EC}$

To prove : ℓ ∥ BC i.e. DE ∥ BC



Proof : If possible, let DE be not parallel to BC. Then, there must be another line parallel to BC. Let DF ∥ BC.

Since DF ∥ BC. Therefore from Basic Proportionality Theorem, we get

$$\frac{AD}{DB} = \frac{AF}{FC} \quad .... \text{(i)}$$

But, $\frac{AD}{DB} = \frac{AE}{EC}$ (Given) .... (ii)

From (i) and (ii), we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AF}{FC} + 1 = \frac{AE}{EC} + 1 \text{ [Adding 1 on both sides]}$$

$$\Rightarrow \frac{AF + FC}{FC} = \frac{AE + EC}{EC}$$

$$\Rightarrow \frac{AC}{FC} = \frac{AC}{EC} \Rightarrow FC = EC$$

This is possible only when F and E coincide i.e. DF is the line l itself. But, DF ∥ BC. Hence, l ∥ BC.

Criteria for Similarity of Triangles

The four important criteria used in determining the similarity of triangles are:-

1. AAA criterion (Angle-Angle-Angle criterion)
2. AA criterion (Angle-Angle criterion)
3. SSS criterion (Side-Side-Side criterion)
4. SAS Criterion (Side-Angle-Side criterion)